

Course: B.Tech.

Branch: Common to all branch

Semester: I

Subject Code & Name: BTBS101 & Engineering Mathematics-I

Max Marks: 60

Date: 16.06.2026

Duration: 3 Hr.

Instructions to the Students:

1. Each question carries 12 marks.
 2. Question No. 1 will be compulsory and include objective-type questions.
 3. Candidates are required to attempt any four questions from Question No. 2 to Question No.6
 4. Use of non-programmable scientific calculators is allowed.
 5. Assume suitable data wherever necessary and mention it clearly
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Q1. Objective type questions. (Compulsory Question)

12

- 1) A non-homogeneous system of linear equations $AX = B$ has a no solution if.....
 - a) $\rho(A) = \rho[A | B]$
 - b) $\rho(A) \neq \rho[A | B]$
 - c) $\rho(A) < \rho[A | B]$
 - d) $\rho(A) > \rho[A | B]$
- 2) The Cayley–Hamilton Theorem states that.....
 - a) every matrix is singular
 - b) every matrix satisfies its own characteristic equation
 - c) determinant of a matrix is always zero
 - d) eigenvalues are always equal
- 3) If $\lambda_1, \lambda_2, \lambda_3$ are the Eigen values of a square matrix A with $|A| \neq 0$ then Eigen values of A^{-1} are.....
 - a) $\lambda_1, \lambda_2, \lambda_3$
 - b) $-\lambda_1, -\lambda_2, -\lambda_3$
 - c) $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$
 - d) $-\frac{1}{\lambda_1}, -\frac{1}{\lambda_2}, -\frac{1}{\lambda_3}$
- 4) If $f(x, y) = xy$ then the value of $\frac{\partial^2 f}{\partial x \partial y} = \dots\dots\dots$
 - a) y
 - b) x
 - c) 0
 - d) 1

- 5) If $z = f(x, y)$ and $x = \phi(u, v)$ and $y = \psi(u, v)$ then which of the following is correct?
- a) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$ b) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{dx}{du} + \frac{\partial z}{\partial y} \frac{dy}{du}$
- c) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} - \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$ d) none of these
- 6) For a homogeneous function $z = x^3 + y^3$, Euler's theorem gives.....
- a) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$ b) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z$
- c) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 3z$ d) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$
- 7) If $r = f_{xx}(a, b)$, $s = f_{xy}(a, b)$, $t = f_{yy}(a, b)$, $rt - s^2 > 0$ and $r > 0$, then the function $f(x, y)$ has.....
- a) local maximum b) local minimum c) saddle point d) no extreme value
- 8) If $J_1 = \frac{\partial(x, y)}{\partial(u, v)}$ and $J_2 = \frac{\partial(u, v)}{\partial(x, y)}$ then $J_1 \cdot J_2$ is equal to
- a) 1 b) -1 c) 0 d) None
- 9) The number of loops of $r = a \sin 2\theta$ is
- a) 6 b) 2 c) 3 d) 4
- 10) If the equation of a curve remains unchanged when x and y are interchanged, then the curve is symmetrical about.....
- a) origin b) X-axis c) $y = -x$ d) $y = x$
- 11) The value of $\int_1^a \int_1^b \frac{1}{xy} dx dy$ is.....
- a) $\log a \cdot \log b$ b) $\log(a \cdot b)$ c) $\log a + \log b$ d) none
- 12) The area bounded by two curves $y = f_1(x)$ and $y = f_2(x)$ intersecting in two points $A(a, b)$ and $B(c, d)$ is.....
- a) $\int_a^b \int_c^d dx dy$ b) $\int_c^d \int_{f_1(x)}^{f_2(x)} dx dy$ c) $\int_a^c \int_{f_1(x)}^{f_2(x)} dx dy$ d) none of these

Q2. Solve the following

A) Find the rank of matrix A by reducing it to normal form where,

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

B) For what value of λ and μ the equations,

$$2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$$

have i) infinite solution, ii) a unique solution, iii) no solution.

Q3. Solve the following.

A) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then show that, $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$.

B) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$ then show that, $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$.

Q4. Solve Any Two of the following.

A) If $x = u(1 - v)$, $y = uv$ prove that $J J' = 1$.

B) Expand $f(x, y) = e^{x+y}$ in Maclaurin's theorem up to fourth term.

C) Find the maximum value of $x^m y^n z^p$ where $x + y + z = a$.

Q5. Solve Any Two of the following.

A) Evaluate $\int_0^1 x^4 (1 - x^2)^{\frac{3}{2}} dx$

B) Trace the curve $y^2 = x^3$ (Semi-cubical Parabola)

C) Trace the curve $r = a(1 + \cos \theta)$

Q6. Solve Any Two of the following.

A) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dx dy dz$

B) Change the order of integration and evaluate $\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}} dx dy$

C) Find the eigen value and eigen vector for least positive eigen value of the matrix,

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$